Numerical modelling of magnesium die-castings using stochastic fracture parameters

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Abstract

Quasi-static material tests using specimens cut from a generic cast component are performed to study the behaviour of the high-pressure die-cast magnesium alloy AM60 under different stress states. The experimental data set is applied to establish a validated probabilistic methodology for finite element modelling of thin-walled die-castings subjected to quasi-static loading. The test specimens are modelled in the explicit finite element (FE) code LS-DYNA using shell elements. The cast magnesium alloy AM60 is modelled using an elasto-plastic constitutive model including a high-exponent, isotropic yield criterion, the associated flow law and isotropic hardening. To simulate fracture, the Cockcroft–Latham fracture criterion is adopted, and the fracture parameter is chosen to follow a modified weakest-link Weibull distribution. Comparison between the experimental and predicted behaviour of the cast magnesium specimens gives very promising results.

1. Introduction

To reduce the fuel consumption, the automotive industry strives for structural solutions combining low weight with low cost. In this respect, the high-pressure die-casting of light weight metals such as aluminium and magnesium alloys has attracted attention as being a competitive production method. One of the main challenges with this production method is to optimise the process parameters with respect to the part design and the solidification characteristics of the alloy in order to obtain a sound casting without casting defects. Unbalanced filling and lack of thermal control can cause bifilms, porosity and surface defects due to turbulence and solidification shrinkage. As a result, stochastic characteristics tend to prevail in the fracture behaviour of the castings. Design and production of thin-walled cast structural components for the automotive industry involve development of alloys and manufacturing processes, structural design and crashworthiness analysis. To reduce the lead time for development of a new product, it is necessary to use finite element (FE) analysis to ensure a structural design that fully exploits the material without sacrificing safety. Accurate description of the material behaviour is essential to obtain reliable results from such analyses. To minimise the weight of the structural component while maintaining the safety in a crash situation, the ductility of the material has to be utilised without risking un-controlled failure. Hence, a reliable failure criterion is also required, enabling the designer to take advantage of the potential of the cast material.

From the literature, it is seen that several different approaches for finite element based modelling of fracture for thin-walled castings have been investigated over the last years. In a work by Ockewitz et al. [1], it has been suggested to use a
material model considering micro mechanical damage such as the Gurson model [2]. Chen et al. [3] predicted fracture using a maximum plastic strain and a maximum principal strain criterion. Their conclusion was that both of the investigated criteria have limitations in prediction of fracture initiation, and that stress state should be accounted for. Altenhof et al. [4] suggested to use a pure stress based criterion. Leppin et al. [5] proposed to use the IDS criterion (Instability, Ductile and Shear fracture) [6], a fracture model that requires a minimum of six different material tests for calibration of an isotropic and homogeneous material. Mae et al. [7] calibrated the Bao–Wierzbicki fracture locus [8] for a cast aluminium alloy by using a total of 12 tests. Another approach for modelling of fracture in high-pressure die-casting aluminium alloys have been explored by Mohr and Treitler [9] by using a phenomenological fracture criterion that was fitted to the experimental results of four different material tests. In previous works [10–13] by the authors of this manuscript, it has been suggested to use the relatively simple phenomenological ductile fracture criterion as proposed by Cockcroft and Latham [14], a criterion that for isotropic and homogeneous materials only requires one single material test to be calibrated. However, as the experimental results from these previous studies have demonstrated, and also as pointed out in the work by Gokhale and Patel [15], the measured ductility can vary significantly due to stochastic variations in sizes, numbers and amount of defects present in the cast material.

Fig. 1 shows the geometry of the generic AM60 component investigated in this study, together with the corresponding gating system. The length of component is 400 mm and the wall thickness is approximately 2.5 mm. In previous studies [10,16], the AM60 material was characterized using uniaxial tensile tests, uniaxial compression tests, and plate bending

**Fig. 1.** Illustration of generic cast component: length = 400 mm, thickness = 2.5 mm, width = 80 mm, and height = 40 mm.
tests. The results from the uniaxial tensile tests showed that the scatter in elongation at fracture is quite large. The poorest area is the outlet side, where values of effective plastic strain at fracture as low as 2–3% were measured. The best areas were found to be the 80 mm flange in front of the gates, where values of effective plastic strain at fracture as high as 22% were measured [10]. An overview of the tensile elongation distribution is provided in Fig. 2. The experimental results also revealed different tensile and compressive behaviour for magnesium alloys. For details, it is referred to Dørum et al. [16]. However, since the reported strength difference is relatively small, this has not been further examined here. The components were cast of magnesium alloy AM60 at Hydro’s Research Centre in Porsgrunn, Norway with a Bühler SC42D 420-ton cold chamber die-casting machine.

A recent study on aluminium die-castings [13] indicated that identification of fracture parameters for cast materials depends significantly on the size and/or the geometry of the material test specimens. In this study, the stress–strain behaviour and fracture characteristics of die-cast magnesium alloy AM60 are investigated. Uniaxial tension tests, plane-strain tension tests, notched tension tests, and shear tests are carried out using specimens cut from generic cast components. This allows studying the influence of specimen geometry (size effects) and stress state on the observed behaviour. The fracture surfaces are investigated using scanning electron microscope (SEM). A new approach for FE modelling of fracture in castings is developed. The material behaviour is described by an elasto-plastic model including a high-exponent, isotropic yield criterion, the associated flow law and isotropic hardening. Fracture is modelled by the Cockcroft–Latham criterion [14], assuming the fracture parameter to follow a modified weakest-link Weibull distribution [17]. Shear fracture (due to shear band localisation) has not been accounted for in the present work.

2. Material tests

Four types of tests were performed for the cast AM60 alloy: uniaxial tension, plane-strain tension, notched tension and shear tests. The different types of tests were carried out using specimens cut from some selected positions in the cast U-profile, as illustrated in Fig. 3. With this procedure, cast material cut from identical positions in the component is subjected to different states of stress and strain.

The tests were carried out in a hydraulic testing machine under displacement control. Force and displacement/strain were continuously measured. The displacement rate was adjusted to obtain a strain rate approximately equal to $2 \times 10^{-3} \text{s}^{-1}$. All tests were carried out at ambient temperature.

Uniaxial tension specimens were cut from the inlet wall and the outlet wall in the longitudinal direction, and from the 80 mm web of the casting in both the longitudinal and transverse direction. The geometry of the tensile specimens is

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**Fig. 2.** Ductility map for cast AM60 component, which illustrates the distribution of effective plastic strain at fracture throughout the component [12].

**Fig. 3.** Position of specimens cut from the generic cast component.
shown in Fig. 4a. The strain in the length direction was measured by an extensometer with 25 mm gauge length. The experimental engineering stress–strain curves for the AM60 casting are shown in Fig. 5, while Cauchy stress versus logarithmic plastic strain curves are provided in Fig. 6 for different parts of the component. It is seen that the difference between tests in the longitudinal and transverse directions is small, both with respect to stress–strain behaviour and ductility. From the tests in the longitudinal direction, it is observed that there are marked differences in ductility with position in the casting, while the stress–strain behaviour is generally much less affected. In particular, the material in

Fig. 4. Material test specimens: (a) uniaxial tension, (b) plane-strain tension, (c) shear, and (d) notched tension.
the outlet flange is less ductile than the material in the inlet flange and in the web. Furthermore, the material in position 1 is more ductile than the material in position 2 for the 80 mm web. The scatter in ductility between duplicate tests is significant, and is caused by casting defects that vary from one tensile test specimen to another in a stochastic manner. For comparison, the engineering stress–strain curve obtained from uniaxial compression test in a previous work [16] is shown together with the data from uniaxial tensile tests using specimens cut from the web in longitudinal direction, see Fig. 5. No significant strength difference is observed.

Plane-strain tension tests were carried out using specimens cut from the 80 mm web of the U-profiles. The specimen is taken in the longitudinal direction, and the geometry is illustrated in Fig. 4b. The longitudinal displacement over the central region was measured by an extensometer with 55.5 mm gauge length. Fig. 7 shows the measured force–displacement curves. There is significant scatter in the ductility for specimens machined from the same position in the castings, i.e. between duplicate tests. With respect to spatial variations, the cast material in position 1 is again more ductile than the material in position 2. The force level is consistent between the duplicate tests, and is not significantly affected by position. The variations in displacement at fracture show that the ductility in cast components can vary from casting to casting, even for castings produced in a well-controlled manner.

Shear tests were carried out using specimens cut from the 80 mm wide web of the U-profiles. The specimen geometry is shown in Fig. 4c. The specimens were aligned with the longitudinal direction of the AM60 components. The longitudinal displacement over the central region of the shear specimen was measured by an extensometer with 55.5 mm gauge length. The measured force–displacement behaviour is provided in Fig. 8. It is seen that there is no significant difference in the force–displacement behaviour and ductility of specimens cut from positions 1 and 2. This could indicate that the cast material
is less sensitive to the spatial distribution of casting defects when subjected to shear loading. However, it should be kept in mind that the volume of strained material is considerably smaller in the shear test than in the uniaxial and plane-strain tension tests.

Notched tension test specimens were cut from the 80 mm web of the casting, in the transverse direction. The geometry of the specimens is shown in Fig. 4d. With respect to possible size effects, the critical area where fracture is likely to initiate is significantly smaller in the notched tensile specimen compared with the critical area for fracture in the ordinary uniaxial tensile specimen (Fig. 4a). However, the state of stress is not dramatically changed, even if some transverse stress will be introduced. The displacement over the notch was measured by an extensometer with 25 mm gauge length. Fig. 9 shows the measured force–displacement curves from the notched tensile tests. Fracture occurs at maximum force. Significant differences are observed in ductility of the cast material with position in the casting, supporting the results from the uniaxial tensile tests. Material cut from position 1 in the 80 mm web is consistently more ductile than the material cut from position 2 when subjected to tensile loading.
3. Metallurgical considerations

The mechanical properties of an alloy depend on the defects that may be present in the matrix. These defects could be point, line, surface or volume defects. Among these defects, volume defects (porosity, secondary phases or inclusions) are known to be the most significant ones and may affect the mechanical properties dramatically.

Inclusions, basically oxides, play an important role in casting operations. Particularly in high-pressure die-casting operations, with casting speeds of minimum 15 m/s up to 40 m/s, the liquid metal advances into the mould in jets that introduce the surface oxide to become incorporated into the melt. However, the oxide inclusions cannot exist in melts as a single, because the only way they can become incorporated into the liquid is by entrainment action [18]. During such a simple folding action, the two non-wetted oxide surfaces come in contact to form a bifilm that acts as a crack in the casting. Therefore, in high-pressure die-castings, the casting will have a spatial distribution of casting defects. The size and population of these defects are critical since they act as the initiation points for porosity and also as stress risers. As seen from Fig. 10, pictures
using scanning electron microscope (SEM) show that the fracture surface has a high density of crack-like pores. By closer examination, it can be confirmed that the crack-like pores are all oxides.

It is well known that in the presence of defects or stress risers, the components may fracture at stresses far away from their nominal theoretical limits. Fig. 6 is a perfect example to such phenomena. Even within the groups (longitudinal and transverse direction, outlet and inlet side) there is a huge scatter of elongation and maximum stress values. It is important to note that even the proof stress changes considerably. Here, the oxides (or bifilms) act as a sort of strengthening mecha-
nism in the matrix which is very similar to the behaviour of metal matrix composites. It is also interesting to note that there is one sample in Fig. 6 that fractures even before reaching the proof stress. This pre-mature fracture is another example of the presence of defects (most probably bifilms) in the casting.

4. Material model and parameter identification

The experimental work by Kelley and Hosford [19] showed that the yield surface of textured magnesium alloys takes complex shapes. Their conclusions were that the investigated polycrystalline materials were very anisotropic and that the anisotropy increased with increasing levels of texture. In addition, the yield surfaces were neither elliptical nor centred at the origin due to the directionality of the twinning mode, resulting in a different behaviour in tension and compression. According to Cazacu and Barlat [20]: if the internal shear mechanism of plastic deformation is sensitive to the sign of the stress, then the macroscopic yield function ought to be represented by an odd function of the principal values of the stress deviator. Lou et al. [21] investigated the hardening evolution of AZ31B sheet. They constructed a model of deformation mechanisms on the basis of predominantly basal slip for initial tension, twinning for initial compression, and untwincing for tension following compression. Staroselsky and Anand [22] developed a crystal-mechanics based model for polycrystalline hcp materials applied to magnesium alloy AZ31B.

Previous work has shown that the fine-grained HPDC AM60 material can be regarded as isotropic and that the difference in tension and compression behaviour is relatively small [16]. Thus, the cast magnesium alloy AM60 is modelled using an elasto-plastic constitutive model including a high-exponent, isotropic yield criterion, the associated flow law and isotropic hardening. It should be noted that the choice of yield criterion is not based on any experimental data in the literature supporting this choice for an hcp material and that the chosen yield surface is not able to describe any asymmetric behaviour (different behaviour in tension and compression). Fracture is modelled by element erosion when a fracture criterion is reached. The model has been implemented in the explicit finite element code LS-DYNA [23].

The high-exponent isotropic yield criterion [24,25] is written in the form

\[ \varphi(\sigma_1, \sigma_2) = (\sigma_1)^{2k} + (\sigma_2)^{2k} + (\sigma_1 - \sigma_2)^{2k} = 2\sigma_Y^{2k} \]  

where \( \sigma_1 \) and \( \sigma_2 \) are principal stresses in plane stress and \( k \) is a material parameter. For \( k = 1 \) the high-exponent yield surface reduces to the classical von Mises yield surface. In this work, the exponent \( k \) is evaluated against the available experimental data. The flow stress \( \sigma_Y \) is defined by the isotropic hardening rule

\[ \sigma_Y = \sigma_0 + \sum_{i=1}^{2} Q_i (1 - \exp(-C_i \varepsilon_e)) \]  

where \( \varepsilon_e \) is the effective plastic strain, \( \sigma_0 \) is the proportionality limit, and \( Q_i \) and \( C_i \) are hardening parameters. Using a least squares method, the hardening parameters were determined from the Cauchy stress versus logarithmic plastic strain curves in Fig. 6 and are given in Table 1. This implies that any variation in flow stress with position in the casting was not accounted for in the FE simulations.

An uncoupled continuous–discontinuous approach to describe fracture is adopted (see e.g. Mediavilla et al. [26]). This means that the influence of damage evolution on the material behaviour is neglected and there is no material softening before initiation of fracture. Crack propagation is described by element erosion when a fracture criterion is fulfilled within the element. The Cockcroft–Latham criterion [14] was used in the simulations, i.e.

\[ W = \int \max(\sigma_1, 0)d\varepsilon_e \leq W_c \]  

where \( \sigma_1 \) is the maximum principal stress and \( W_c \) is the critical value of the integral \( W \). Hence, fracture occurs when \( W = W_c \). Henceforth, \( W_c \) will be referred to as the fracture parameter, while \( W \) will be denoted the Cockcroft–Latham integral. It is seen that fracture cannot occur when the maximum principal stress is compressive and that neither stresses nor strains alone are sufficient to cause fracture. Furthermore, the fracture strain increases with decreasing stress triaxiality (in the shear tests, the stress triaxiality is significantly reduced compared to the uniaxial tension test).

As the Cockcroft–Latham fracture criterion is based upon only one parameter, a single material test is sufficient for the calibration. In the present study, the fracture parameter \( W_c \) was independently identified from uniaxial tension tests, plane-strain tension tests, notched tension tests, and shear tests. In this way, the validity of the Cockcroft–Latham fracture criterion for different deformation modes (or stress states) and statistical effects on fracture may be evaluated. If the Cockcroft–Latham fracture criterion accurately describes fracture for cast magnesium alloys, one would expect that the value of

<table>
<thead>
<tr>
<th>Alloy</th>
<th>( \sigma_0 ) (MPa)</th>
<th>( Q_1 ) (MPa)</th>
<th>( C_1 )</th>
<th>( Q_2 ) (MPa)</th>
<th>( C_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM60</td>
<td>30.3</td>
<td>106.1</td>
<td>1169.7</td>
<td>241.9</td>
<td>8.8</td>
</tr>
</tbody>
</table>
the fracture parameter would be approximately the same for all of the tests. However, as the cast material is very inhomogeneous, due to the spatial distribution of casting defects, it was expected that size effects could be quite significant.

The uniaxial tensile test specimens failed before the point of diffuse necking for the AM60 alloy, and, accordingly, the stress and strain fields are uniform up to fracture. Hence, the fracture parameter is obtained as the area under the work-hardening curve, since for uniaxial tension, the Cauchy stress \( \sigma \) equals the maximum principal stress \( \sigma_1 \) and the logarithmic plastic strain \( \varepsilon_p \) is equal to the effective plastic strain \( \varepsilon_e \). The resulting values of \( W_c \) are listed in Table 2 for the inlet flange, the outlet flange and the web. The dramatic variation in ductility is reflected in the large variations of \( W_c \) with position but also between uniaxial tension tests at the same position. For the other types of tests, the stress and strain fields are not uniform and the fracture parameter \( W_c \) has to be estimated via FE analysis. To this end, each of the material tests was simulated numerically without accounting for fracture. With this procedure, the value of the Cockcroft–Latham integral \( W \) could be monitored throughout the gauge area and the fracture parameter could be identified for the investigated material tests. It was assumed that \( W_c \) is equal to the maximum value of \( W \) within the specimen’s gauge area at maximum load.

To evaluate the shape of the yield surface, the material tests were simulated with different values of the material parameter \( k \), defining the shape of the yield surface. It is noticed that \( k = 1 \) gives the von Mises criterion, while \( k \to \infty \) results in the Tresca criterion. Here, simulations were run with \( k = 1 \) and 4. Shell (or membrane) elements were used to model the specimens, since plane stress prevails in all test specimens. Homogeneous material properties were assumed for the various parts of the cast material, adopting the hardening parameters given in Table 1. The deformation of the specimens was applied smoothly but much faster than in the experiments. It was checked that the kinetic energy and its variation were both negligible throughout, and thus the simulations can be considered quasi-static.

The FE mesh of the plane-strain tension test specimen consists of 26,644 shell elements, giving a characteristic element size of 0.6 mm in the gauge area. A comparison between the experimental and predicted force–displacement behaviour is provided in Fig. 11. It is seen that force–displacement behaviour is well described in the simulation. Better agreement with the experiments is obtained with \( k = 4 \), but the influence of this parameter is small. Also shown in Fig. 11, is the maximum value of the Cockcroft–Latham integral \( W \) over the specimen’s gauge area. The dashed vertical lines in the figure indicate the displacement at fracture in two of the tests, while the corresponding horizontal dashed lines point towards the estimated values of \( W_c \). The calculated fracture parameters are listed in Table 2. It is seen that the uniaxial and plane-strain tension tests give fracture parameters of the same order for the web of the U-profile, although the fracture parameters for the plane-strain tension test is somewhat higher.

**Table 2**

Cockcroft–Latham fracture parameters \( W \) (MPa).

<table>
<thead>
<tr>
<th></th>
<th>Uniaxial tension test</th>
<th>Plane-strain tension test</th>
<th>Shear test</th>
<th>Notched tensile test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inlet wall</td>
<td>Web</td>
<td>Outlet wall</td>
<td>Web</td>
</tr>
<tr>
<td></td>
<td>7.9–22.0</td>
<td>9.3–44.6</td>
<td>0.5–6.5</td>
<td>25–57</td>
</tr>
</tbody>
</table>

![Fig. 11. Comparison of the experimentally measured and numerically predicted force (F/A₀) together with the predicted Cockcroft–Latham integral W as functions of displacement (w) for plane-strain tension tests for AM60.](image)
The shear test specimens were modelled by 8675 shell elements. The characteristic element size in the gauge area is 0.1 mm. The predicted force–displacement curve and the maximum value of the Cockcroft–Latham integral versus displacement are illustrated in Fig. 12. As for the simulation of the plane-strain tension test, \( k = 4 \) gives excellent agreement with the experimental results. The estimated fracture parameters \( W_c \) are listed in Table 2, where it has been assumed that fracture occurs at peak force. By comparing the values of the fracture parameter obtained from the shear tests with those obtained from the uniaxial and plane-strain tension tests, it is seen that the former values are about three times higher for AM60. This indicates that the Cockcroft–Latham criterion is not generally valid when assuming homogeneous material properties. Furthermore, the fracture mode may change depending on the material test; uniaxial tension, plane strain tension, notched tension and in-plane shear. However, it should be recalled that the volume of material tested in the shear tests is only a small fraction of the volume tested in uniaxial and plane-strain tension. Thus, owing to statistical effects, the probability of testing a part of the material having a defect of a given size and orientation is much smaller in the shear tests, which should lead to increased ductility. It can therefore be assumed that the maximum \( W_c \) value obtained from these shear tests represent a good estimate on the maximum ductility that can be achieved in a flawless material.

The notched tensile test specimen was modelled using 720 shell elements, giving a characteristic element size of 0.8 mm. Fig. 13 shows a comparison of the experimental and numerical force–displacement behaviour and the predicted maximum value of the Cockcroft–Latham integral \( W \) versus displacement. The agreement is again excellent, and it is observed that the shape of the yield surface seems to be of little importance. The estimated values for \( W_c \) are compiled in Table 2. Somewhat unexpectedly, the maximum value of \( W_c \) in the notched tensile tests is a bit higher than the maximum value obtained in the shear tests. Regarding transferability of damage parameters between the different types of material tests, is should be noted that the geometry of the notched tensile specimen was chosen to give a stress–strain condition in the narrowed section of the specimen which was similar to uniaxial tension. Thus, the increased values for \( W_c \) estimated from this test compared to those obtained from uniaxial tensile tests indicated that size effects are important with respect to calibration of damage parameters for the investigated material. Another approach to investigate the size effect could be taken by comparing tensile and shear tests having the same volume of material in the gauge area in which fracture can take place. However, this would require that the gauge area were uniformly strained in both the uniaxial tensile test and the shear test. For the uniaxial tensile test specimen used in this work, the volume of the gauge area is equal to \((10.3 \times 25 + 10.5 \times 6) \times 2.5 \text{ mm}^3 = 801 \text{ mm}^3\). For the in-plane shear test, the material in the narrowed section of the specimen is not uniformly strained. But the volume in which fracture is likely to initiate is less than 5 mm (length of the narrowed section) \( \times \) 2 mm (approximately width of narrowed section) \( \times \) 2.5 mm (thickness) = 25 mm³. As an illustration of the volume where fracture may initiate, Fig. 14 shows contour plots of the integral \( W \) at the point of fracture for the most ductile of the different specimens. It should be noted that due to the non-homogeneity of the microstructure, the most critical element may not necessarily be located in the region with the highest values for \( W \).

5. A probabilistic approach to fracture modelling

By comparing the values of the fracture parameter obtained from the notched tensile tests and the shear tests with those obtained from the uniaxial and plane-strain tension tests, it is seen that the former values are significantly higher. This could
mean that the Cockcroft–Latham criterion is not valid, or it could be an evidence of a size effect, since the specimens of the
two groups of tests have different gauge areas. Owing to statistical effects, the probability of testing a part of the material
having a casting defect of a given size and orientation is much smaller when the gauge area is small, which should lead
to increased ductility. It is therefore probable that the differences in fracture parameters obtained from the different tests
are (at least partly) due to a size effect.

Zhou and Molinari\[27,28\] propose a micro-cracking model for brittle materials (ceramics) considering the stochastic dis-
tribution of internal defects. The model introduces a Weibull distribution of the local strength of cohesive elements. Thus,
the probability of introducing a weak cohesive element increases with the cohesive element size. Inspired by this idea, the
fracture parameter $W_c$ of a finite element is assumed to follow a modified weakest-link Weibull distribution in the current
study.

The Weibull distribution\[17\] gives the fracture probability $P(\sigma)$ of a material volume under effective tensile loading, i.e.

$$P(\sigma) = 1 - \exp \left[ -\left( \frac{\sigma}{\sigma_0} \right)^m \right]$$

(4)

where $V$ is the volume, $V_0$ is the scaling volume, $\sigma_0$ is the scaling stress, and $m$ is the Weibull modulus. Since cast magnesium
alloys are not brittle materials, the use of a critical fracture stress is not justified. Instead, the Cockcroft–Latham ductile frac-
ture criterion is adopted, and the fracture probability of a material volume is recast as

$$P(W) = 1 - \exp \left[ -\left( \frac{W}{W_0} \right)^m \right]$$

(5)

where $W_0$ is the scaling value for the fracture parameter. Consequently, the corresponding probability density function for the local material ductility reads

$$f(W_e) = \left. \frac{dP}{dW} \right|_{W=W_e} = m \frac{V}{W_0} \left( \frac{W_e}{W_0} \right)^{m-1} \exp \left[ -\left( \frac{W_e}{W_0} \right)^m \right]$$

(6)

In the finite element model, the fracture parameter is assumed to follow the modified weakest-link Weibull distribution, rep-
resented by Eqs. (5) and (6). The volume $V$ is then the volume of the element, which implies that the ductility of an element
decreases with increasing size. Hence, large elements are more likely to fail than small elements since they have higher prob-
ability of containing defects. In practice, the Weibull distribution of the fracture parameter is achieved by using a random
number generator and inverse transform sampling. A random number $u$ is generated from the standard uniform distribution.
A trial value of the fracture parameter for element $e$, denoted $W^e_0$, is first calculated such that $P(W^e_0) = u$. As mentioned ear-
lier, it is assumed that the maximum value of $W_e$ obtained from the shear tests represents a reasonable estimate of the max-
imum ductility $W^{\text{max}}_e$ of the cast material. Thus, the local value of the fracture parameter for element $e$ is taken as $W^e_e = \min(W^e_0, W^{\text{max}}_e)$. Now, the fracture parameters which are determined from the experimental results represent macro-
scopic fracture of the cast material. To avoid simulating micro-cracks when the element size is small, a length scale has to be
introduced. To this end, the fracture parameter $W_c^*$ is taken as the minimum value of the fracture parameter for all elements within a given radius $L$ emanating from the centre of actual element.

6. Numerical study

The Weibull parameters were calibrated using fracture parameters obtained from the uniaxial tension tests. The scaling volume is taken as the volume of the gauge region of the specimen, i.e. $V_0 = 30 \text{ mm} \times 5 \text{ mm} \times 2.5 \text{ mm}$. The shape of the Weibull distribution is very sensitive to the parameters. With a relatively low number of experimental parallels, as in this work,
the shape of the Weibull distribution depends on the amount of experimental data. To circumvent this problem, it was assumed that the experimentally obtained fracture parameters follow a normal distribution. By first calculating the mean value and the standard deviation of $W_c$, the Weibull parameters $m$ and $W_c$ could then be calibrated to give the best fit to this normal distribution. Table 3 shows the Weibull parameters for the inlet wall, the outlet wall and the 80 mm web of the generic cast AM60 component. Finally, the non-local radius $L$ was set to 0.5 mm in all simulations to avoid simulating microcracks.

Table 3
Weibull parameters.

<table>
<thead>
<tr>
<th>$V_0$ (mm$^2$)</th>
<th>$m$</th>
<th>$W_c$ (MPa)</th>
<th>$W_{c}^{\text{max}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet wall</td>
<td>150</td>
<td>3.45</td>
<td>16.8</td>
</tr>
<tr>
<td>Web</td>
<td>150</td>
<td>2.49</td>
<td>27.0</td>
</tr>
<tr>
<td>Outlet wall</td>
<td>150</td>
<td>2.29</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Fig. 15. Comparison of the experimentally measured and numerically predicted (six parallels) engineering stress–strain curves for uniaxial tensile tests.

Fig. 16. Comparison of the experimentally measured and numerically predicted force–displacement curves for plane-strain tension tests.
To validate the proposed probabilistic approach for modelling of fracture in magnesium die-castings, the uniaxial tensile tests, plane-strain tension tests, shear tests and notched tensile tests with specimens cut from the 80 mm web were simulated in LS-DYNA. The uniaxial tension test specimens were modelled by 720 shell elements (i.e. a characteristic element size equal to 1.0 mm), the FE-models for the other test specimens were as described in Section 4. Six repeated simulations were carried out for the uniaxial tension tests, plane-strain tension tests and shear tests, while eight parallels were carried out for the notched tensile tests. In each simulation, a different distribution of the fracture parameter was obtained as explained above. The results from the simulations are illustrated in Figs. 15–18 together with the experimental results. As seen in Fig. 15, the predicted engineering stress–strain curves from simulations of the uniaxial tensile tests are very similar to the experimental one. Furthermore, both the slight increase in ductility (in terms of $W_c$ values) for the plane-strain test and the large increase in ductility for the shear tests and the notched tensile tests are captured in the numerical predictions. For the simulations of the shear tests, fracture occurred when the fracture parameter reached the maximum value $W_c^{\text{max}}$ in four of the simulations. Consequently, the predicted scatter is very small, similar to what was observed from the experimental tests. It should be noted that also in one of the simulations of the notched tensile tests, fracture occurred due to the max-

![Fig. 17. Comparison of the experimentally measured and numerically predicted (six parallels) force–displacement curves for shear tests.](image1)

![Fig. 18. Comparison of the experimentally measured and numerically predicted (eight parallels) force–displacement curves for notched tensile tests.](image2)
The distribution of $W_c$ values from the numerical simulations of uniaxial tensile tests that predicted the lowest and the highest value of elongation at fracture is illustrated in Fig. 19a and b, respectively. To examine the mesh sensitivity of the proposed approach to modelling of fracture in high-pressure die-castings, the notched tensile tests were simulated numerically with two refined meshes in addition to the simulations presented in the previous section. The notched specimens were modelled with characteristic element size equal to 0.4 mm and 0.2 mm. For comparison, the notched uniaxial tensile test was modelled with characteristic element size equal to 0.8 mm. Thus, three different meshes were used in the mesh sensitivity study. By comparing the predicted force–displacement behaviour from these simulations, illustrated in Fig. 20, with the predictions shown in Fig. 18 (element size equal to 0.8 mm) it is seen that the predicted response (and the scatter) is very similar for all of the chosen mesh densities. Based on this rather limited mesh sensitivity study, it appears that the proposed stochastic fracture modelling approach is rather mesh size insensitive.

7. Concluding remarks

The quasi-static behaviour of high-pressure die-cast magnesium alloy AM60 has been studied. Experiments were performed with specimens of different geometry and size of the gauge region. The experimental data were used to develop a validated probabilistic methodology for finite element modelling of thin-walled die-castings subjected to quasi-static loading. The main results are summarised in the following:

1. The ductility of the specimens cut from the castings depends on the position in the casting. There are also significant variations in ductility when comparing the measured characteristics of specimens cut from different castings cast under equal casting conditions. Thus, as a result of unstable flow of the liquid magnesium in the mould cavity, the mechanical properties of the casting are considered to be of stochastic nature.
2. An elastic–plastic model including a high-exponent, isotropic yield criterion, the associated flow law and isotropic hardening, was used to model the cast material, and was shown to provide good predictions of the force–displacement behaviour for the investigated material tests.

3. The fracture parameter for the Cockcroft–Latham criterion was identified independently from different material tests. It was found that the identified values varied significantly. The fracture parameters determined from shear tests and notched tensile tests were significantly higher than those identified from uniaxial and plane-strain tension tests. The ductility increased with decreasing volume of the gauge region of the test specimen. This indicates that size effects are significant in calibration of fracture models for die-castings.

4. By combining the Cockcroft–Latham fracture criterion and the Weibull statistical distribution function, the fracture parameter was defined as a stochastic Weibull distributed parameter. Repeated numerical simulations of the material tests were carried out, giving predictions very similar to the experimental behaviour.

5. Simulations of notched tensile tests with FE-models of different mesh densities predicted very similar force–displacement behaviour. Thus, the proposed stochastic fracture modelling approach seems to be relatively mesh size independent.

Accurate numerical prediction of the mechanical capacity (especially in terms of ductility) of castings requires that the inhomogeneous distribution of defects is included. To provide more details about the fracture mechanisms, possible shear band formation could be investigated by optical micrographs of the investigated materials. A coupling of die-casting process simulations and the current approach should be investigated to establish a deterministic-stochastic approach that can model both the variations in ductility depending on the material’s position in the casting as well the stochastic aspects.

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